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Clubs in an Urban Setting

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In this paper the properties of a congestable concentrated local public good (CCoLPG) are described, taking into account both locational aspects and congestion. This discussion, therefore, completes the analysis of congestable local public goods (LPG) which was started in our earlier paper (Congestable local public goods in an urban setting, *J. Urban Econ.*, 290–310 (1982). It is here shown that the optimal provision of CCoLPG leads to the creation of clubs, where the government has to determine the location and provide the optimal quantity of LPG in each club and also levy congestion tolls on users according to their level of utilization of the club. Congestion tolls cover the αest part of the costs, where α is the level of congestability of the CCoLPG. The rest of the costs are covered by taxing land rents. Clubs will form, possibly more than one per city, even if the LPG is pure (i.e., $\alpha = 0$) and they will contain a single household if the LPG is private (i.e., $\alpha = 1$). The difference between these results and those of classical club theorists stem from the fact that here we also take into account the limited supply of land of given accessibility (i.e., locational aspects), which like congestion lead to the creation of clubs. When these two factors are considered together, it explains why congestion tolls alone are not sufficient to finance clubs' operations, or why pure LPGs also have to be provided locally, etc.

This paper is the complement of a previous one written by the author [12]. In the previous paper congestable local public goods (LPG) were defined, characterized, and sorted into two spatial types. One type, the congestable dispersed local public good was studied in a framework of a spatial general equilibrium model of a city and its spatial characteristics were identified and partially analyzed. Optimal behavior of a local government with respect to policies concerning congestable dispersed local public goods were described.

In this paper, the characteristics of a congestable concentrated local public good (CCoLPG) are analyzed in a framework of a general spatial equilibrium model of a city.

A concentrated LPG is one which is located at specific locations and households have to travel to this location in order to consume the LPG and spend time and other resources there. Since the first part of the earlier paper [12] deals also with the nonspatial characteristics of this type of LPG, we will not dwell on these issues here, but proceed immediately to the spatial model, utilizing results of the previous paper.

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It is shown in this paper that optimal provision of CCoLPGs in a city with a homogeneous population leads to the creation of clubs with disjoint residential rings of club members. Samuelson's rule for optimal allocation of public goods, modified to account for congestion, should hold for the capacity of the LPG in each club.

Congestion tolls, equal to the damages caused to club members by the marginal unit of utilization, should be imposed on each unit of utilization by the household of the LPG. Total congestion tolls cover a fraction α of total costs of the LPG, where α is the measure of congestability. The rest of the resources needed to cover the local government expenditure on public goods is available from land rents. For given boundaries between clubs, the optimal location of the LPG is where total travel costs to it are minimized. Given the location of the LPG and its quantity in each of these locations, the boundaries between clubs are determined competitively, and are the locations in which a household is indifferent as to which club it belongs to. Within those boundaries we may find households belonging to the same population group, facing identical product prices, including the price of land, and having the same income and yet having different consumption baskets. The existence of many types of CCoLPG consumed by households, such as parks, schools, swimming pools, etc., each of them having its own set of clubs covering the city, may explain the variety of consumption baskets we encounter in real life, without having to resort to the assumption of differences in household tastes.

Clubs may result from pure concentrated local public goods, i.e., the degree of congestability, α, is equal to zero, and in that case no congestion tolls are paid and the sole source of financing is land rents, or they may result from CCoLPGs in which α is positive. As α increases, all other parameters being kept the same, club sizes reduce and their numbers increase. When α equals one, the CCoLPG becomes a private good. That is, when $\alpha = 1$, in each club there is only one household, and total costs are completely paid by congestion tolls. In this case, the LPG can be provided by private markets.

We should at this point stop to compare our results with those of the classical club theorists. Buchanan first introduced the theory of clubs in 1965 [3], and further developed it in his paper with Goetz [4]. The Berglas paper [2] is but a sample of his extensive and impressive contribution to the subject. In club theory, congestion alone causes the creation of clubs, and the existence of limited accessible land is ignored. Hence, pure public goods, for example, never lead to the creation of clubs in the classical theory, but do so in our case. The club size in the classical theory is fully determined by congestion and it is determined so that optimal congestion tolls fully cover costs. When more than one type of club is involved, income transfers between the two types of club are necessary. This last result can be found not only among club theorists but also among others dealing in LPG like [5] and [19]. As argued in general in [10], this property of fiscal dependence is a result of a distribution of property rights dependent on the city of residency (as in [19]) or on membership in a club (as in club theory). As soon as these nonrealistic assumptions are dropped and land as a marketable property is introduced, fiscal independence is attainable. For further details see [10]. Other differences between results here and those of club theorists are due to the introduction in our model of limited space with a given accessibility. Club theorists have so far ignored this "local" property of the LPG and concentrated only on congestion. Here we take account of both properties and, through changes in the level of congestability, show how the trade-off between locality, represented by land and land rents, and congestability represented by congestion factors and congestion tolls, is worked out.

It is further argued here that regulating housing sizes is not effective as a congestion control measure in the case of CCoLPG as it is in the case of a CDiLPG (see [12]). The reason for this is that now congestion is caused by the utilization level, and not by the household itself, as it was in the previous case. In real life we encounter examples of CCoLPG in which congestion tolls are used as congestion controls.

A particular type of CCoLPG, schools, is then considered. Following an argument of Mills [15] that efficiency requires levying of congestion tolls, possibly in the form of tuition fees, it is speculated why local governments do not, although they have a strong interest in doing so. It is pointed out that optimal tolls will be high in low-income areas and low in high-income areas. It is then argued that such a seemingly regressive tax cannot be imposed by a government wishing to be re-elected. This is so, even though those taxes do not and can not change income distribution. It is then argued that an efficient system of busing can achieve efficiency just as well as congestion tolls.

Thus it is argued that busing, besides being intended to achieve social integration, is also in practice a tool to achieve economic efficiency.

The plan of the paper is as follows: The model's assumptions are discussed in section one. In Section II the efficient solution is derived and the optimal local government behaviour rules are derived and discussed in the framework of a decentralized solution. Alternative local governments' policies intended to control congestion are discussed in Section III.

1. THE MODEL ASSUMPTIONS

Consider a city located around a center 0. Let x measure the distance from the center and let $A(x)$ be the amount of land available for use at x. In a circular city $A(x) = 2\pi x$, and in a linear city $A(x)$ is constant and does not vary with distance. Assume further that industry is located in the central business district (CBD) around the center. All travel in the city is made along the radius towards or away from the center. Travel along a circumference of a circle is costless.

Also assume that transportation requires only income inputs and is not subject to congestion and commuting to the CBD is made all the way to the center. These are accepted simplifying assumptions in urban economics literature when the focus of the study is not on transportation problems. In our model, since travel is made not only to the CBD, the assumptions are even more limiting than usual. Let $t(x)$ be commuting cost to the CBD of a household living at x and let $b(y)$ be the cost to a household travelling a distance y to the location of the CCoLPG such as travel to the school, to a nearby park or museum, etc. Thus we have

$$\begin{aligned} t(0) &= 0, \qquad \dot{t}(x) > 0, \qquad \ddot{t}(x) \le 0, \\ b(0) &= 0, \qquad \dot{b}(x) > 0 \qquad \ddot{b}(x) \le 0. \end{aligned} \tag{1}$$

A dot above a function designates differentiation with respect to distance. Let x_0 be the CBD limit, then the CBD area G is given by

$$G(X_0) = \int_0^{X_0} A(x)\,dx. \tag{2}$$

Two goods are produced in the CBD. One is the city's export good which has an exogenously given price P. The other good is the local public good which is then distributed in the residential area in predetermined locations. Let $\bar{c}$ designate the total amount of public good produced in the CBD, Q the amount of export good produced there and N designate the total labor force employed in the city. Then[1]

$$\begin{aligned} &Q = F(\bar{c}, N, G), \\ &F_{\bar{c}} < 0, \qquad F_N, F_G > 0, \qquad F_{NN}, F_{cc}, F_{G\bar{c}} < 0. \end{aligned} \tag{3}$$

The function F is the transformation function between $\bar{c}$ and the export good.

We assume a single population group in the city whose utility level is equal everywhere due to free migration.

Let U_0 be that overall utility level, then

$$U(h, z, q) = U_0, \tag{4}$$

where $U(\cdot)$ is the utility function of the population group; h, is the amount

[1] If ψ is a function and x, y variables, then $\psi_x = \partial\psi/\partial x$ and $\psi_{xy} = \partial^2\psi/\partial x\,\partial y$. If numerical indexes replace the literal ones, it designates differentiation with respect to the variable whose order equals the number. Thus $\psi_1(x, y) = \psi_x$, $\psi_{12}(x, y) = \psi_{xy}$, etc.

of housing consumed by the household; z, is the amount of all other private consumption goods measured in units of income and q, is the amount of services obtained from a local public good consumed by a household.

Let the local public good be a CCoLPG (see [12, part I]).

The quantity, q, of services of CCoLPG available to the household is produced by a household production function from a public good with capacity C_i available to the household and provided by the government, by the household's level of utilization of the public good, μ, where μ is measured in income units and is a decision variable of the household, it also depends on the household congestion factor k. As discussed in the case of CCoLPG, in [12], the utilization level of the public good μ consists mainly of the household's time. Therefore, it is appropriate to use as the congestion factor the total amount of utilization of the public good by the households rather than the household's relative utilization. To see that clearly, suppose the public good is a park. A household spending time in the park causes a disturbance which equals the disturbance caused by two households, each staying half the length of time in the park. The congestion factor is therefore the same for all households using a given facility j and shall be designated k_j.

If the LPG is congestable of degree α, then;

$$q(x) = q\left(\frac{c_j}{k_j^{\alpha}}, \mu(x)\right) \quad X_{2j-2} < x < X_{2j},$$
$$q_1, q_2 > 0. \tag{5}$$

For further details on the degree of congestability and (5) in general, see [12]. It should be noted that c_j and k_j are exogenously given to the household and are equal for all households using facility j, while $\mu(x)$ is a decision variable of the household and may vary from household to household depending on its location. As we shall see later, the household does affect k_j, however this effect is an external one and the household is not aware of it.

The residential ring is located around the CBD (see Fig. 1). In it the units of capacity of the CCoLPG, c_j, $j = 1, 2 \ldots$, are located at predetermined locations x_{2j-1}. Assume further that the CCoLPG does not require land in its location in the residential ring (this being a simplifying assumption).

The locations x_{2i}, $i = 1, 2 \ldots$, are the boundaries separating households consuming the public goods located at X_{2i-1} and households consuming the public good located at X_{2i+1} (see Fig. 1).

If a household resides at x and consumes public goods at X_i, it has to travel $|x - X_i|$ distance to the location of the LPG and it costs him $b(|x - X_i|)$ to do so. The function $b(y)$ was defined in (1). Housing is

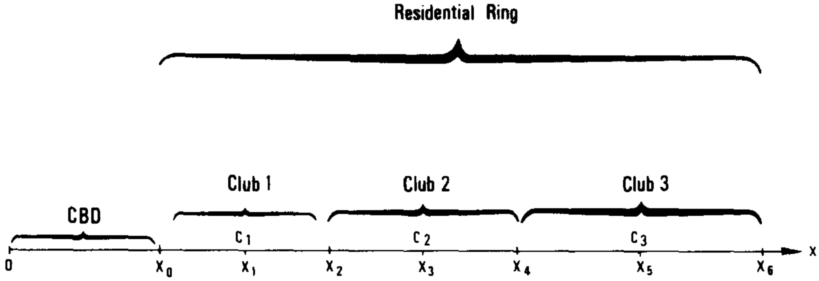

FIGURE 1

assumed to be produced with land only and consumed where the land is located. This is the customary simplifying assumption accepted in the literature. The variable z which represents the quantity of all other private goods consumed by the household is assumed to have a unit price, i.e., z is the good serving as a numeraire.

The number of households per unit land at x is therefore, $(1/h(x))$. The congestion factor of a facility j, in the case of a CCoLPG is the total amount of utilization of the facility by its users. Total utilization of facility j is thus

$$k_j = \int_{X_{2j-2}}^{X_{2j}} \frac{A(y)\mu(y)}{h(y)}\,dy. \tag{6}$$

DEFINITION 1. Following Buchanan, we term facility j of the CCoLPG located at X_{2j-1} as *club j*. The users of the facility are the *club members*, and the area occupied by the facility is the *land of the club* (in our case, equal to zero by assumption). The area in which the club members reside is termed the *residential ring of the club* (*CRR*).

Thus CRR_j is the area between X_{2j-2} and X_{2j}, and N_j, the population residing there form the members of club j. Hence

$$\mathrm{CRR}_j = \int_{X_{2j-2}}^{X_{2j}^0} A(x)\,dx \tag{7}$$

and

$$\text{(a) } N_i = \int_{X_{2i-2}}^{X_{2i}} (A/h)\,dx,$$

$$\text{(b) } N = \Sigma N_i. \tag{8}$$

The total quantity of the public good produced $\bar{c}$ is distributed completely to the different clubs; hence, since a unit of capacity is equal to unit quantity (see [12] for further discussion):

$$\bar{c} = \sum_{i=1}^{\infty} c_i. \tag{9}$$

Let S be the net city surplus. Then

$$\begin{aligned} S = {} & PF(\bar{c}, N, G(X_0)) + NV \\ & - \sum_{i=0}^{\infty} \left(\int_{X_{2i}}^{X_{2i+1}} \frac{A(x)}{h(x)} (z + t(x) + \mu + b(X_{2i+1} - x)) dx \right. \\ & \left. + \int_{X_{2i+1}}^{X_{2i+2}} \frac{A(x)}{h(x)} (z + t(x) + \mu + b(x - X_{2i+1})) dx \right) \\ & - R_A \int_0^{X_\infty} A(x) dx. \end{aligned} \tag{10}$$

Maximization of S given by (10) subject to (4), (5), (6), (8), and (9) with respect to variables c_i, $\bar{c}$, N, X_i, k_j, $z(x)$, $\mu(x)$, and $h(x)$ is a necessary and sufficient condition to local efficiency of the city, which is in turn a necessary condition for Pareto optimality of the whole economy. The idea behind this is that if all surpluses in the economy, including S, are not at their optimal value, somebody in the economy can be made better off without making anyone else worse off. This can be done by increasing S subject to the constraints and giving this newly created surplus to somebody in the economy. For further details see [10, 12].

II. THE EFFICIENT SOLUTION

The following are necessary conditions of the optimization problem formulated at the end of the last section. They are obtained by performing the necessary derivations, eliminating the shadow prices by substitution and then by performing some additional manipulations of the equations.

$$-PF_c = \int_{X_{2j-2}}^{X_{2j}} (A/h) \cdot (U_q/U_z)(q_1/k_j^\alpha) dx, \qquad j = 1, 2 \ldots, \tag{12}$$

$$1 + (-PF_c)\frac{\alpha c_j}{k_j} = \frac{U_q}{U_z} q_2;\ X_{2j-2} < x < X_{2j}, \qquad j = 1, 2 \ldots, \tag{13}$$

$$\begin{aligned} & z(x) + t(x) + \mu(x) + b(|X_{2i-1} - x|) + \frac{U_h}{U_z} \cdot h \\ & + \alpha(-PF_c)\frac{c_i}{k_i}\mu - PF_N - V = 0, \\ & X_{2i-2} < x < X_{2i}, \qquad i = 1, 2 \ldots. \end{aligned} \tag{14}$$

The following propositions can now be stated:

PROPOSITION 1. *In the (Pareto) optimal solution the rate of product transformation (RPT) between the public good and the private export good, evaluated in terms of the private consumption good, when the LPG is pure, i.e.,* $\alpha = 0$, *is equal to the sum of the marginal rates of substitution in consumption (RCSs) between the public good and the private consumption good of all members of the same club.*

Thus, Samuelson's well-known rule for efficient allocation of resources between private and public goods *holds for the capacity and consumers of each club separately.*

Proposition 1 follows immediately from (12), taking into account that $A(x)/h(x)$ is the number of households in location x and $(U_q/U_z)q_1$ is the marginal rate of substitution between the public and the private good. In our model we have only one population group, and hence all households are identical. The above rule can probably be extended to heterogeneous populations. However, the investigation of this possibility must be postponed to another study.

PROPOSITION 2. *When the degree of congestability is greater than zero, each household gets services only from* $(1/k_j^\alpha)$ *of the total capacity of the* LPG, *and the marginal rate of product transformation (RPT) between the public and the private good is equal only to* $(1/k_j^\alpha)$ *of the sum of marginal rates of substitution in consumption (RCSs) between the public and private good of club members, where* k_j *is the congestion factor.*

Proposition 2, like Proposition 1, follows from (12).

Note that, when $\alpha = 1$, the LPG is actually divided between consumers and each household gets only one share of it, C_j/k_j. The RPT between the public and private good is now equal to an average of the sum of RCSs. This is not necessarily the case with a private good, in which RPT should equal the RCS of each single household, but the case of a private good certainly fulfils this condition.

Propositions 1 and 2 are the equivalent of Propositions 1 and 2 in [12]. Hence, if we define the case of CDiLPG as dealing with clubs of unit area and CRR, *the relations in Propositions 1 and 2 hold for these types of clubs as well.*

PROPOSITION 3. *In the case of a decentralized allocation of resources,*[2] *in order to achieve efficiency, a congestion toll, equal for all members of the same club, but different for members of other clubs, has to be levied on club members*

[2]By a decentralized allocation is meant a competitive market equilibrium in which the government's activities are confined to taxation, subsidization, or the provision of public goods. See [12] for further details.

per unit of utilization. This toll is given by (15), *where j is the club index and T_j is the congestion toll per unit of utilization to members of this club*

$$T_j = -PF_c \frac{\alpha c_j}{k_j}. \tag{15}$$

The toll T_j is equal to the marginal damages to club members caused by an additional unit of utilization.

To see this, let $R(x)$ be the land rent in a decentralized solution. Then, in the residential ring where households equate marginal rates of substitutions in consumption to price ratio, we have

$$R(x) = U_h(x)/U_z(x). \tag{16}$$

Since utilization is measured in units of income, its price is equal to one. Hence, in order to make (13) hold in a decentralized allocation, a congestion toll, $T_j(x)$ (as in (15)) per unit of μ has to be levied. By substituting (15) and (16) into (14), and noting that PF_N, the value of the marginal productivity of labor, is equal to the equilibrium wage rate, (14) becomes the budget constraint of a household located in x. This proves Proposition 3.

PROPOSITION 4. *Total optimal congestion tolls paid by members of club j equal the αest part of the cost of the capacity of the* CCoLPG *in the club*,[3] *where α is the degree of congestability.*

Obviously, when $\alpha = 0$ no congestion exists and therefore no congestion tolls are paid, since the good is a pure LPG. When $\alpha = 1$, congestion tolls cover the total cost of the facility.

The rest of the optimality conditions, besides (12)–(14), concern the determination of the values X_i, $i = 1, 2, \ldots$. The following variables have to be defined before those conditions can be listed.

DEFINITION 2. A household located at X_{2j} can belong to either club j or club $j+1$. If the household in X_{2j} belongs to club j, we designate its consumption basket by an upper index[1], i.e., $(h^1(X_{2j}), z^1(X_{2j}), q^1(X_{2j}))$,

[3] Total congestion tolls paid by users of the j's facility are

$$\int_{X_{2j-2}}^{X_{2j}} \frac{A}{h} T_j \mu \, dx = \int_{X_{2j-2}}^{X_{2j}} \frac{A}{h} \alpha(-PF_c)\frac{c_j}{k_j}\mu \, dx$$
$$= \alpha(-PF_c)\frac{c_j}{k_j}\int_{X_{2j-2}}^{X_{2j}} \frac{A}{h}\mu \, dx = \alpha(-PF_c)c_j, j = 1, 2, \ldots.$$

The last step in obtaining the above equation is derived by substituting (7) into the equation. The term $(-PF_c)c_j$ on the extreme right-hand side of the above expression is the total cost of the facility in club j. This proves Proposition 4.

and if it belongs to club $j+1$, we designate its consumption basket by an upper index^2, i.e., $(h^2(X_{2j}), z^2(X_{2j}), q^2(X_{2j}))$. In the same way, $U^i(X_{2j})$ is the utility function of a member of club $j+i-1$, $i=1,2$, living in X_{2j}.

By the index $i=L$ it is meant that club L is the last club in the city, i.e., for all $i>L$, $X_{2i-1}=X_{2i}=X_{2L}$, and $C_i=0$.

The rest of the necessary conditions can now be expressed by[4] (17)–(20).

$$\int_{X_{2i-2}}^{X_{2i-1}} \frac{A(x)}{h(x)} b_1(X_{2i}-x)dx - \int_{X_{2i-1}}^{X_{2i}} \frac{A(x)}{h(x)} b_1(x-X_{2i})dx = 0, \quad (17)$$

$$U_h^1(X_{2i})/U_z^1(X_{2i}) = U_h^2(X_{2i})/U_z^2(X_{2i}), \quad (18)$$

$$PF_G = R(X_0), \quad (19)$$

$$R_A = R(X_{2L}). \quad (20)$$

Equations (19) and (20) are the usual relations determining the CBD and city limits.

PROPOSITION 5. The spatial equilibrium condition of clubs.

The optimal location of the CCoLPG *in club* j, X_{2j-1}, *when the club limits* X_{2j}, X_{2j-2} *are given, is such that it minimizes total travel costs to it by club members. To achieve this, the location of the* LPG *must be such that a marginal shift of it in any direction will cause a zero marginal change in total travel costs to it.*

The intuition behind this last rule is clear. If, due to a marginal change in the location of the LPG, *total travel costs change, say, decrease, then by shifting the location of the* LPG *in the same direction, we can save costs, which implies that the present location is not optimal. If the change in total costs is positive, then a shift of the* LPG *in the opposite direction will save travel costs, again indicating that the* LPG*'s location is not optimal. Only if no change in travel costs occurs due to a marginal shift can the* LPG*'s location be optimal.*

The proof of Proposition 5 follows directly from (17), in which the left-hand side measures the marginal change in total travel costs to the location of the LPG, due to a marginal movement of this site away from the center.

PROPOSITION 6.[5] *The rates of substitution in consumption between housing and the composite consumption good of two households located on the border between two clubs must be equal, even if each household belongs to a different club. Since, in a competitive market, those marginal rates of substitution are equal to the land rent in this location, we can conclude that market competition*

[4]See Appendix 1 for details on how these last equations are obtained.

[5]The proof follows directly from (18).

will determine the optimal border between clubs without government intervention.

The above argument does not mean, however, that consumption baskets of two such households are the same. On the contrary, it is quite likely that h^1 and h^2, z^1 and z^2 and q^1 and q^2 are all different from each other. Thus we may find two households belonging to the same population group (by the same population group we mean that the two households have identical tastes, skills and nonearned income) and residing in the same location are yet dwelling in houses of different sizes and consuming different quantities of other goods. If, however, they belong to the same club, their consumption baskets must be identical. The existence of many types of such CCoLPGs with different boundaries and club sizes, may explain much of the diversity of consumption baskets of households belonging to the same population groups we find in real life.

PROPOSITION 7.[6] *When the degree of congestability, α, increases, and every other parameter is kept unchanged, the CRR_j, the capacity C_j of the* LPG *and the number of members in each club decreases and the number of clubs in the city increases. When α is equal to one, each household is a club of its own and consumes its own private* LPG.

PROPOSITION 8.[7] *If the* LPG *is pure, or close to it (i.e., α is very small), then C_j, the capacity of the club, decreases with j (i.e., with distance from the centre) and the area of the CRR_j increases with j.*

[6]The first part of the proposition follows when we note that, in (12), for a given $\bar{c}$, if α is increased while the right-hand side of (12) is kept constant, c_j, k_j and CRR_j must decrease. The quantity of LPG saved this way is then used to construct the additional clubs now needed to provide services to the whole city. By so doing, travel costs to the LPG are saved, thus increasing total city surplus. Therefore, the solution with the decreased CRR and more clubs is superior to the one optimal for a smaller α. Consequently, as α increases optimal clubs shrink in size and membership. When $\alpha = 1$ if we substitute $k_j = \mu_j$ and $\mathrm{CRR}_j = h(X_{2j})$, we see that all the necessary conditions still hold. Furthermore if, in the latter case, the number of households in each club exceeded one, nothing would be gained in services of LPG to the households and extra costs would be added. This makes the single household clubs case superior to any other solution and hence optimal. Q.E.D.

[7]The following argument is intended to show that, if the LPG is a pure public good or almost so (i.e., α very small), then c_j decreases with j and the differences $|X_{2j} - X_{2j-2}|$ increase with j. The reason is that as one gets further away from the centre, the density of population decreases and there are less households per unit of land at a given distance from the facility which can use it. To keep the right-hand side of (12) unvarying with j, the relation $(U_q/U_z)q_1$ should increase with j. Since housing becomes cheaper with distance relative to z, the consumption of z decreases with distance, hence U_z increases. This implies that $U_q \cdot q_1$ must increase everywhere at a faster rate than that of U_z. This can be achieved by letting c_j/k_j^{α} decrease with j. Since α is very small, the effect of k_j^{α} is negligible, hence c_j itself must decrease. In the same way, the area of each CRR_j, i.e., the area between X_{2j-2} and X_{2j} must increase with j as well in order to increase efficiency. If $\alpha = 1$ then c_j is a private good and all we can

In this model, land at the consumption sites of the LPG has not been used as input in the production of the LPG's capacity. Should land be an important factor in the production of the capacity of the public good, as is often the case in practice, for example in recreation areas, parks, etc., then the cost of producing the CCoLPG reduces as we get further away from the CBD due to reduction in the price of land and it is likely that then efficiency would dictate an increase in c_j as we get further away from the centre. Furthermore, it is interesting to note that this also implies that the ratio of land to all other production factors always increases as we get further away from the centre in the production of the LPG facilities.

PROPOSITION 9.[8] *When $\alpha = 1$, the LPG is a private good and can be supplied by private markets.*

PROPOSITION 10. *Let TR be the total land rents in the city and TR_A the total alternative agricultural value of the urban land, then*[9]

$$TR - TR_A = S + (1 - \alpha)(-PF_c)\bar{C}. \tag{21}$$

Equation (21) implies that in the optimum, since the city surplus s^* is nonnegative,[10] land rents are sufficient to finance the part of the expenditure on the public good not covered by congestion tolls.

PROPOSITION 11.[11] *An aposteriori criterion exists which helps the local government judge the desirability of its actions concerning the provision of CCoLPG. There are two types of actions involved in this case. The choice of location of the CCoLPG, which does not involve government spending, and the provision of the LPG in each of these locations. If, due to the government action, total urban land rents increased by more than the government expenditure on this action, then the government action is justified. If, however, expenditure exceeded the increase in land rents, then the government action contributed to a decrease in efficiency and is therefore undesirable and should be avoided.*

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conclude in this case is that the consumption of z and q together is reduced as we get further away from the centre due to substitution effects.

[8] The proof follows from Propositions 4 and 7. The first implies that the household pays, by congestion tolls, the entire price of the public good it uses, and the second implies that its share of the LPG is consumed only by the household itself. Thus private markets could sell the quantity required by each household for its market price g, and the solution obtained in this way is identical to the optimal solution described here.

[9] See Appendix 2 for proof of (21).

[10] Note that a city of zero size and surplus is in the feasible set of the optimization problem. Since S^* cannot be exceeded by any surplus from the feasible set, $S^* \geq 0$.

[11] The proof of proposition 11 again follows from (21) and bearing in mind that an action that increases S contributes positively to efficiency and hence is desirable. This rule is part of a more general one. For details see [10].

The optimum is reached when the government expenditure on a marginal change in the location of the LPG or in the quantity of LPG provided there is matched by exactly the same change in land rents.

III. MECHANISMS FOR CONTROLLING CONGESTION

The only way to achieve efficiency is to control the source of the congestion which, in our case, is the amount of utilization of the CCoLPG by the households. It can be done either through congestion tolls or by direct regulation of each household's level of utilization μ. The control of any other variable such as number of households per unit land (which is equivalent to controlling lot size) would not produce efficiency. This was proven in general by Baumol and Oates [1], and can be easily verified directly in our specific case. *Controlling the level of utilization through regulation is impossible in practice in most cases and when it is possible* policing costs are often very high. Thus the advantages of zoning regulations discussed in [12] do not hold in this case. Regulating lot size might be a second best solution if the optimal policy cannot be pursued. In practice we find that this second best possibility is used quite often. It might be interesting to find out how much we can improve welfare by this second best solution compared with no control of congestion at all on the one hand and the optimal control on the other, but this is beyond the scope of this paper.

Congestion tolls, in the form of entrance fees, are in many cases of CCoLPG easy and therefore relatively cheap to collect. The CCoLPG is usually a single facility occupying a limited space which can be easily fenced. Indeed, in many cases of such goods entrance fees are collected. Simple entrance fees may not always suffice, since different households may spend different amounts of time in the facility. Usually, however, the difference in the length of stay is not significant. When it is significant, measures can be devised to account for it as well.

For example, in amusement parks, we encounter the phenomenon of charging entrance fees and fees for rides on each of the facilities as well. In tollways travellers have to pay at regular intervals in which gates are located or, to determine payment, use is made of a punched card which indicates where they entered and left the road. A similar arrangement exists in parking lots and other places.

When the use of the CCoLPG includes the use of special equipment, which contributes to added congestion, it is customary to charge entrance fees for this special equipment as well. These charges also serve as congestion tolls. For instance, consider a lake beach where entrance fees are charged. Special fees exist for boats, tents, and even fishing rods. When one rents a boat on the beach instead of bringing his own boat, congestion tolls are included in the renting fee.

Thus we see that arrangements take care of congestion problems in many instances. The congestion tolls are then passed on to the municipality through licensing fees, business taxes, or land taxes.

In spite of the above we do find a typical CCoLPG, operated by local authorities, which is often congested and yet no congestion tolls or other obvious efficiency controls exist. The case of neighborhood schools is discussed by Mills [15] and local governments are strongly criticized by him for their reluctance to impose effective congestion control measures, such as school fees for each child. The ease of imposing, collecting, and policing such fees is apparent. In [11] is also argued that the local government has a strong incentive to levy such a toll, yet there is no evidence that any local government has ever tried seriously to impose such a tax.

The apparent reason for this is that such a tax would seem to be aimed against low income households. Congestion problems in schools become more acute as the density of population increases. Thus in suburban areas, where the rich live, it is likely that efficiency dictates that no congestion tolls should be levied or, if some are in order, they would be very low, compared to the tolls a household in the central city, where the poor live, will be asked to pay. It should be pointed out at this stage that if such tolls were imposed by the local government they could not have any effect on the well-being of residents. It could only affect land values in the city. If it was imposed optimally, net land values would increase throughout the town, *especially* in the central city. Yet, like the effect of welfare payments, discussed in [11], the real effect is contrary to common convictions. Indeed, such a tax, newly imposed, could in the very short run decrease the well-being of central city residents, but as soon as it becomes anticipated (in the intermediate and long run) it will have no effect on the well-being of residents. (For a discussion of local government motivations, and long- and short-run effects of local government policies, see [11].) Unlike the problem of welfare payments, which can be avoided passively by the city government not performing the wrong policy, in this case of schools, the local government has to perform an unpopular act in order to achieve efficiency. Apparently very few governments were able to do so in the past, and it is doubtful whether they would be able to do so in the future.

Nevertheless, something can be done and has been done. Instead of charging central city students tuition, suburban students can be subsidized. Moreover, there is no need to subsidize all suburban students. It is sufficient to subsidize a marginal fraction of the student population. This subsidy can be in the form of busing pupils from the central city to schools in the suburban areas. High-quality busing services from home and better education available in the suburbs may be sufficient to achieve the same effect as congestion tolls.

The above arguments imply that, although busing is considered to be a tool to increase integration, it is also useful for increasing economic efficiency and, because it is an efficiency tool, there is a good chance that it will be successful and soon become a way of life.

APPENDIX 1

Equations (17)–(20) in the text are a result of the differentiation of S, defined by (10), subject to the constraints given by (4), (5), (6), (8) and (9) with respect to $X_j, j = 0, 1, 2, \ldots$.

Equation (17) is the result of differentiating with respect to X_{2i-1}, $i = 1, 2, \ldots$, and then eliminating the Lagrange multipliers from the solution.

By differentiating the Lagrangian with respect to X_{2j}, $j = 1, 2, \ldots$, we obtain, after eliminating from the result the Lagrange multipliers,

$$\sum_{i=1}^{2} \frac{(-1)^{i-1}}{h^i(X_{2j})} \{ z^i(X_{2j}) + t(X_{2j}) + \mu^i(X_{2j})(1 + T_{j+i-1}) + b((-1)^{i-1}(X_{2j} - X_{2j+2i-3})) \} = 0, \quad j = 1, 2, \ldots . \tag{A1}$$

(18) in the text is obtained after substituting (14) and then (15) into (A1).

By differentiating with respect to X_0, and then eliminating the Lagrange multipliers we obtain,

$$PF_G h(X_0) + z(X_0) + t(X_0) + \mu(X_0)(1 + T_1) + b(X_1 - X_0) - PF_N - V = 0. \tag{A2}$$

By substituting (12) into (14) and the result for $x = X_0$ in (A2), we obtain (19). When taking into account that for all $j > 2L$, $X_j = X_{2L}$, and then differentiating with respect to X_{2L}, we obtain

$$PF_N + V - z(X_{2L}) - t(X_{2L}) - \mu(X_{2L}) - b(X_{2L} - X_{2L-1}) - R_A h(X_{2L}) - \mu(X_{2L}) \int_{X_{2L-2}}^{X_{2L}} \frac{\alpha A}{h} \frac{U_q}{U_z} \frac{q_1 c_L}{k_L^{1+\alpha}} dx = 0. \tag{A3}$$

Substituting (12) into (14) and the result for $x = X_{2L}$ into (A3) yields (20).

APPENDIX 2

The total net gains of the industry are absorbed in the long run by land rents in the CBD. Let TR_C be those rents, then

$$TR_C = PF(\bar{c}, N, G) + (-PF_c) \cdot \bar{c} - PF_N \cdot N. \tag{A4}$$

Let TR_R be the total residential urban rents, then

$$TR_R = \int_{X_0}^{X_{2L}} A(x)R(x)dx. \tag{A5}$$

By substituting (16) into (14) and solving $R(x)$ from the result and substituting it in (A5), we obtain the following expression for TR_R

$$TR_R = \sum_{i=0}^{L-1} \int_{X_{2i}}^{X_{2i+1}} \left(\frac{A(x)}{h(x)} \left(PF_N + V - z(x) - t(x) - \mu(x) - b(|X_{2i+1} - x|) - \alpha(-PF_c)\frac{C_{i+1}}{k_{i+1}}\mu(x) \right) \right) dx. \tag{A6}$$

Let TR_A be

$$TR_A = R_A \int_0^{X_{2L}} A(x)dx \tag{A7}$$

and let

$$TR = TR_C + TR_R. \tag{A8}$$

Substituting (A4), (A6), (A7) and (A8) into (10) results in (21).

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